

# A Nonlinear Dynamic Strategy for Mathematical Modeling and Simulation of Stabilized Platform in Planar Motion in One Body and Three Bodies

**Babak Rohani<sup>1</sup>, Emre Akgul<sup>2</sup>, Mehmet Mutlu<sup>3</sup>, Afsar Saranlı<sup>4</sup>,  
Yigit Yazicioglu<sup>5</sup>**

<sup>1</sup> *Middle East Technical University Department of Mechanical Engineering,  
Turkey* [babak.rohani@metu.edu.tr](mailto:babak.rohani@metu.edu.tr)

<sup>2</sup> *Middle East Technical University Department of Electrical and Electronics  
Engineering, Turkey* [emrea@eee.metu.edu.tr](mailto:emrea@eee.metu.edu.tr)

<sup>3</sup> *Middle East Technical University Department of Electrical and Electronics  
Engineering, Turkey* [memutlu@metu.edu.tr](mailto:memutlu@metu.edu.tr)

<sup>4</sup> *Middle East Technical University Department of Electrical and Electronics  
Engineering, Turkey* [afsars@metu.edu.tr](mailto:afsars@metu.edu.tr)

<sup>5</sup> *Middle East Technical University Department of Mechanical Engineering,  
Turkey* [yigit@metu.edu.tr](mailto:yigit@metu.edu.tr)

Extended Abstract

## 1 Introduction

An operating plant can be subjected to disturbances varying from small vibrations due to its internal structure to large drifts or oscillations that may exist in the environment. Especially in the moving vehicles these disturbances are almost unavoidable. Depending on the intensity of such disturbances some applications that should be performed on the plant might become impossible to do. In order to be able to do the work, stabilization platforms are used to compensate or at least to reduce the intensity of disturbances that are on the body.

The stabilization platforms are mostly designed to carry various types of devices such as sensors or robotic actuators. The effect of disturbances on the plant can be catastrophic for the device itself or its purpose. For instance, cameras mounted on aircrafts are subjected to large disturbances, however, with the use of stabilization platforms they can capture sharp images without any motion blur even for low height flying aircrafts under turbulent air conditions [1]. Also in an underwater suspended work platform a stabilization platform helps to control the working process in an efficient way even though the underwater work platform is subjected to large water drifts [2]. For stabilization platforms one of the the most widely used mechanism is Stewart Platform which is mainly referred as hexapod.

The mentioned platform is a parallel kinematic structure with 6 degrees of freedom. The mechanism is consist of a stationary platform and mobile platform that are connected via six struts mounted on universal joints[3].

In this paper, firstly a novel configuration is presented as a stabilization platform. The mentioned stabilization platform is designed to carry a camera on a legged hexapod robot RHex [4] since images captured by an onboard camera on RHex exhibit extremely large motion blur. By stabilizing the camera platform, motion blur is desired to be canceled or at least reduced to some safe amount.

Stabilization of platform in planar pitch motion is subjected to this paper. In this manner only one actuator is needed, however, the planar joint between vertical column and platform allows the platform to rotate only in pitch degree of freedom.

For stabilization of platform mathematical model of platform is presented, such that the angular changes in platform in pitch degree of freedom is related to the motor shaft displacement.

The equation of motion analysis for platform is derived for one-body and three-body by using Lagrange equations. For one-body analysis the mass properties of platform are assigned and in three-body case mass properties of platform, motor body and motor shaft are subjected to the Lagrange equation. Only the main results are presented in extended abstract and the detailed derivations will be in the main paper.

Finally, the obtained mathematical models are simulated. In simulation part the presented model is simulated by MATLAB Simulink. Simulation results for kinematic analysis and force analysis due to one-body are shown. For 3-body analysis main formulation of velocity and angular velocity of bodies and equation of motion are given. In the main text simulation results for 3-body setup will be presented and the result of this section will be compared with one body analysis results.

## 2 Kinematic Analysis

In Fig. 1 the stabilization platform in planar motion together with body fixed reference frames is shown.

For calculating the relation between motor shaft displacement and platform angular displacement the equations

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OE} + \overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CB}, \quad (1)$$

$$a\vec{j} + b \cos(\theta)\vec{i} + b \sin(\theta)\vec{j} = e\vec{i} + d\vec{j} + c \cos(\alpha)\vec{i} + S \cos(\alpha)\vec{j} - S \sin(\alpha)\vec{i} \quad (2)$$

should be satisfied.

By defining the intermediate parameters;

$$A = e^2 - c^2 + (a - d)^2 + b^2, \quad (3)$$

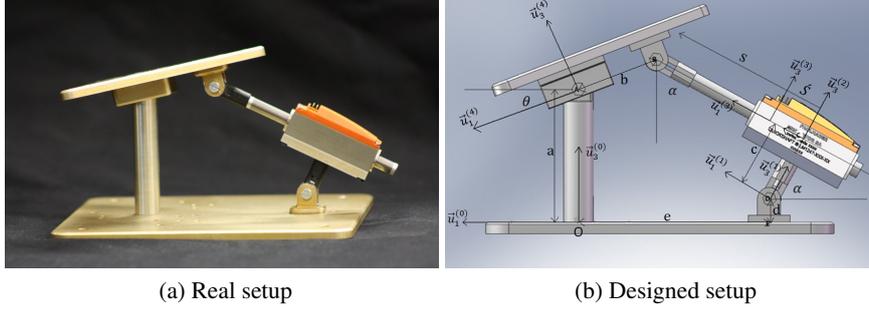


Figure 1: Illustration of reference frame and other parameters in proposed setup

$$, B = 2eb. \quad (4)$$

$$C = 2b(a - d). \quad (5)$$

the equation

$$S^2 = A - B \cos(\theta) + C \sin(\theta) \quad (6)$$

is obtained. Taking square root of the equation (6), the relation between angular displacement of platform and the actuator displacement

$$S = \sigma \sqrt{A - B \cos(\theta) + C \sin(\theta)}, \sigma = \pm 1. \quad (7)$$

is derived.

$\sigma = -1$  is impossible because there is joint restriction. So,  $\sigma = +1$  is the only physically meaningful solution.

### 3 Simplified Equation of Motion (Platform Dynamic Model) Using Lagrange Equation (1-Body)

First we start by neglecting all masses other than the platform mass in order to obtain simplified dynamic model. Lagrange equation procedure is used for dynamic analysis that assigns to the platform via linear motor.

In the analysis it is assumed that there are no dry or viscous friction between the mechanical parts, so that the dissipation energy change is equal to zero, i.e.  $\frac{\partial D}{\partial \theta} = 0$ . Also, the potential energy change is zero, because mass center of platform is fixed, i.e.  $\frac{\partial U}{\partial \theta} = 0$ . Skipping the derivations which will be in the main paper, the Lagrange equation

$$\dot{P}_\theta - \frac{\partial K}{\partial \theta} + \frac{\partial D}{\partial \dot{\theta}} + \frac{\partial U}{\partial \theta} = Q_\theta \quad (8)$$

is used to analyze the proposed system. So, the Equation of motion

$$J_{p22}\ddot{\theta} = F \frac{B \sin(\theta) + C \cos(\theta)}{2S} \quad (9)$$

is obtained. Also the actuating force expression is

$$F = J_{p22}\ddot{\theta} \frac{2\sqrt{A - B \cos(\theta) + C \sin(\theta)}}{B \sin(\theta) + C \cos(\theta)}. \quad (10)$$

#### 4 Platform Dynamic Model Using Lagrange Equation (3-Body)

In this section we extend the result of the previous section by taking into account all relevant masses in the relevant mechanical setup. These are platform, motor body and motor shaft masses. Because the additional masses are placed of center with respect to the rotational motion, variable potential energy is introduced as well as kinetic energies due to the translational velocities. We proceed as follows

From Fig. 1b displacement of the motor mass center

$$\vec{r}_m = \vec{r}_{m/o} = \overrightarrow{OC} = \overrightarrow{OE} + \overrightarrow{ED} + \overrightarrow{DC} \quad (11)$$

gives us the scalar amount of motor velocity final result is obtained as follows;

$$\vec{v}_m^2 = c^2 \dot{\alpha}^2. \quad (12)$$

The angular velocity of the motor mass center is obtained as follows;

$$\vec{\omega}_m = \vec{\omega}_{1/0} = -\dot{\alpha} \vec{u}_2^{(1)} = -\dot{\alpha} \vec{u}_2^{(2)}. \quad (13)$$

Displacement of the motor shaft mass center

$$\vec{r}_{sh} = \vec{r}_{\dot{B}/O} = \overrightarrow{OB} = \overrightarrow{OE} + \overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CB}. \quad (14)$$

that gives the scalar motor shaft velocity

$$v_{sh}^2 = (c\dot{\alpha} - \dot{S})^2 + (\dot{S}\dot{\alpha})^2. \quad (15)$$

The angular velocity of the motor shaft mass center is obtained as follows;

$$\vec{\omega}_m = \vec{\omega}_{1/0} = -\dot{\alpha} \vec{u}_2^{(1)} = -\dot{\alpha} \vec{u}_2^{(2)} = -\dot{\alpha} \vec{u}_2^{(3)}. \quad (16)$$

The kinetic energy equation

$$K = \frac{1}{2} m_m c^2 \dot{\alpha}^2 + \frac{1}{2} J_{m22} \dot{\alpha}^2 + \frac{1}{2} [(c\dot{\alpha} - \dot{S})^2 + (\dot{S}\dot{\alpha})^2] + \frac{1}{2} J_{sh22} \dot{\alpha}^2 + \frac{1}{2} J_{p22} \dot{\theta}^2 \quad (17)$$

and the potential energy expression

$$U = (d + c \sin(\alpha))m_m g + [d + c \sin(\alpha) + (S - 59 \cos(\alpha))]m_{sh} g \quad (18)$$

are calculated.

Finally the equation of motion from Lagrange procedure

$$\begin{aligned} F \frac{B \sin(\theta) + C \cos(\theta)}{2S} = & m_m c^2 \ddot{\alpha} + J_{m22} \ddot{\alpha} + m_{sh} c (c \ddot{\alpha} - \ddot{S}) \\ & + m_{sh} (2\ddot{S} \dot{\alpha} + \dot{S}^2 \ddot{\alpha}) - m_{sh} (c \ddot{\alpha} - \ddot{S}) \\ & + m_{sh} (2\dot{S} \ddot{\alpha} + \dot{\alpha}^2 \ddot{S}) + J_{sh22} \ddot{\alpha} + J_{p22} \ddot{\theta}. \end{aligned} \quad (19)$$

## 5 Simulation Based Validation of the Dynamic Model

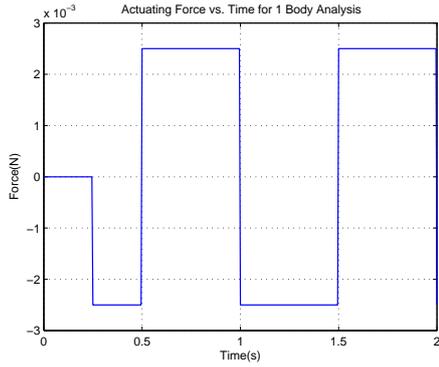


Figure 2: Force applied to the platform by the actuator

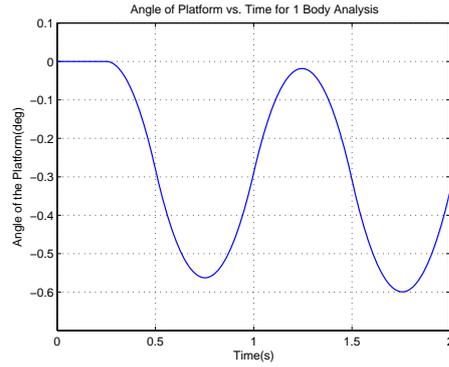


Figure 3: Angular displacement of the platform due to the applied force

In this section we simulate the formulated equation of motion for one-body and present the result. However the 3-body results will be presented in the main text. Our aim is first to observe a reasonable model behavior and, second to observe the approximation error of one-body model in comparison to 3-body model. Our ongoing work also aims to experimentally validate both models using data from the physical setup. This will also result in matching values for model parameters.

Fig. 2 illustrates the applied force by the actuator and Fig. 3 shows the corresponding angle of the platform of the one-body model. Although this simulation does not cover all performance criteria, it shows the validity of performed analysis for one-body.

## References

- [1] M. Baumker, F-J. Heimes, H. Hahn, W. Klier, R. Brechtken, and T. Richter. Mathematical modeling, computer simulation, control and applications of a stabilized platform of an airborne sensor. In *Proceedings of the 19th International Society for Photogrammetry and Remote Sensing (ISPRS) Congress, The Netherlands, Amsterdam. p. 278-86*, 2001.
- [2] R.V. Bostelman and J.S. Albus. Stability of an underwater work platform suspended from an unstable reference. In *OCEANS '93. Engineering in Harmony with Ocean. Proceedings*, pages II321 –II325 vol.2, oct 1993.
- [3] D. Stewart. A platform with six degrees of freedom. *Proc. Inst. Mech. Eng.*, 180, 1965/1966.
- [4] U. Saranli, M. Buehler, and D. E. Koditschek. Rhex: A simple and highly mobile hexapod robot. *International Journal of Robotics Research*, 20(7):616–631, 2001.